Class 8: 09/29/10

§18: Monadic Predicates and Open Sentences

§19: The Existential Quantifier

§20: The Universal Quantifier

§21: Further Notes on Paraphrase

§22: Universe of Discourse
We can take any statement of the form, “The sun is bright” and replace the subject with a placeholder:

“① is bright”

• “①” marks the empty place that can be filled with any name:
  - The sun
  - The flashlight
  - The glare...

• “① is bright” is a monadic predicate: an expression that contains a placeholder and which becomes a statement when the placeholder is replaced with the name of an object.
• Because monadic predicates are not statements, they are neither true nor false.
• But monadic predicates are true or false of particular objects:
  
  “① is bright”

• We can form all sorts of monadic predicates:
  - “① is French”
  - “Everyone who know ① fears ①”
  - “① is tall”
• Some monadic predicates will be true of all objects.
  “一号 is the same thing as 一号”

• Some monadic predicates will be true of no objects.
  “一号 is an even prime number greater than 2”

• **Extension**: The class of objects of which a monadic predicate is true.
  “一号 is a planet in the solar system larger than Earth”

• Two or more predicates may have the same extension: they are **coextensive**.
  “一号 directed Hellraiser”
  “一号 directed Midnight Meat Train”
• The truth-value of sentences constructed from monadic predicates depend only on the extensions of the monadic predicates.
• So we can replace any monadic predicate with a coextensive one without affecting its truth value.

• For now, we can put a variable in place of the placeholder:

   “x is bright”

• “x is bright” is not a statement: it is an open sentence.
• Assigning some object to “x” makes the open sentence true just in case the monadic predicate is true of the individual assigned as value to “x”.

• We can construct complex open sentences using logical operators:

   “x is a director · x is Canadian”
   “x is a director ⊃ x is Canadian”

• A complex open sentence may contain more than one variable:

   “x is a director ⊃ y is Canadian”
We use the **existential quantifier** \((\exists)\) to indicate the existence of something:

\[
(\exists x)(x \text{ is a cat } \cdot x \text{ likes cheese})
\]

- “There is an object \(x\) such that \(x\) is a cat and \(x\) likes cheese.”
- “There is an object such that the object is a cat and the object likes cheese.”
- “There is an object that is a cat and likes cheese.”
- “A cat that likes cheese exists.”
- “Some cat likes cheese.”
- “Some cats like cheese.”
• “There is,” “there are,” “exists,” and “some” can all be paraphrased with “∃”.

When is “(∃x)(x is a cat • x likes cheese)” true?

How about: “There is an unhappy cat”?

“(∃x)(x is a cat • ¬(x is happy))”

How about: “There are some cats who either like cheese or are unhappy”?

• “Some cats like cheese or aren’t happy.”

“(∃x)(x is a cat • (x likes cheese v ¬(x is happy)))”
Sometimes in logical paraphrase, we may need to specify whether we’re talking about a cat, or a person, or whatnot, and sometimes not.

We can compound existential quantification truth functionally:

“(∃x)(x is a cat · x likes cheese) · (∃x)(x is a cat · x is happy)”

• This is not equivalent to:
  “(∃x)(x is a cat · x likes cheese · x is happy)”

• Whatever is enclosed by the parentheses following the “∃” is called the scope of the quantifier.
• If “x” occurs in the scopes of different quantifiers, we treat it like different variables.

• “(∃x)” binds the variable “x”.

• If a variable appears which is not bound by the quantifier, then it is a free variable and can be assigned values.
“(∃x)(Fx)”

- x is F.
- x has the property F.
- x belongs to F.

- “F” (or any other variant) used this way is called a monadic predicate letter.
  - “F” could stand for any number of things:
    - “...is fat”, “...is fragrant”, “...is flatulent”

- So “(∃x)(Fx)” reads: “There exists some x such that x is F.”
• “(∃x)(Fx)” is true just in case there is at least one x and that x indeed has the property “is F” (belongs to the class of things that are F).

• We might use

  “(∃x)[Cx · (Lx v ¬(Hx))]”

to mean “There exists something such that that thing is a cat and it either likes cheese or is not happy”... or simply “Some cats like cheese or aren’t happy.”

- When will this be true?
“∀x(Fx)”

- "∀" is the **universal quantifier**.
- “For all x, x is F.”
- “(∀x)(Fx)” is true just in case every assignment of a value to x makes “Fx” true.
  - In other words, “(∀x)(Fx)” is true iff everything that exists has the property “is F”.

“(∀x)(x is animal v x is vegetable v x is mineral)"

- How do we translate this?
- When is it true?
• Just like the existential quantifier, the universal quantifier has a **scope** and **binds** the variable within the bounds of the scope.

“\((\forall x)(x \text{ is animal} \lor x \text{ is vegetable} \lor x \text{ is mineral})\)”
“\((\forall x)(x \text{ is animal}) \lor (\forall x)(x \text{ is vegetable}) \lor (\forall x)(x \text{ is mineral})\)”

• Try: “All unicorns have horns.”

**NOT**: “\((\forall x)(x \text{ is unicorn} \cdot x \text{ has a horn})\)”
**BUT**: “\((\forall x)(x \text{ is unicorn} \supset x \text{ has a horn})\)”
Paraphrasing from ordinary English can get difficult.

- Not every sentence that implies that use of a universal quantifier will say “every” or “all”...
  - “Unicorns have horns”; “Cats chase mice”; “Girls aren’t allowed”
    “(∀x)(x is a girl ⊃ ¬(x is allowed))”
  - “Only boys are allowed” would be paraphrased:
    “(∀x)(x is allowed ⊃ x is a boy)”
• Universally quantified conditional have the following property: **If no value for “x” makes the antecedent of the conditional true, then the universally quantified conditional is true.**

• Unlike (∃x), (∀x) does not necessarily assume any such x exists.
  - So it can deal with unicorns, sisters that I have, horses in this room...
• Universal quantifications can be truth functionally compounded—
  - With other universal quantifications:
    “(∀x)(Ux ⊃ Hx) ⊃ (∀y)(Ly ⊃ Ny)”
    ➢ “If every unicorn has a horn, then every leprechaun is nervous.”

  - With existential quantifiers:
    “(∀x)(Lx) ⊃ (∃x)(Ox)”
    ➢ “If everybody won the lottery, then somebody owes me money.”
• I could have made “\((\forall x)(Lx) \supset (\exists x)(Ox)\)” more explicit:

“\((\forall x)(Px \cdot Lx) \supset (\exists x)(Ox)\)”

➢ “For every x, if x is a person and x won the lottery...”
➢ But since only persons can win the lottery, this much is assumed.

• This is called the **universe of discourse**—the range of quantifiers—and we can use it to restrict the range of discussion.
• Paraphrasing can be tricky, can seem really clumsy, and takes a lot of practice.

• Sometimes we can use either “(∀x)” or “(∃x)” — particularly in cases of negation.

• “There are no small parts”:

  “(∀x)(Px ⊃ ¬Sx)”

  “¬(∃x)(Px ⋅ Sx)”

  - “Not all” is equivalent to “some not”.

• So yes, we can negate “(∀x)” and “(∃x)”.

  - In general, “(∃x)” is equivalent to “¬(∀x)¬”, and “(∀x)” is equivalent to “¬(∃x)¬”.
• When a statement in ordinary English contains both a quantifier and a negation, it can be unclear whether the negation lies **within the scope** of the quantifier, or if it negates the whole quantified statement.

• How should we paraphrase: “Darren did not fail all of his exams”?

  Option 1: “(∀x)(Ex ⊃ ¬Fx)”
  Option 2: “¬(∀x)(Ex ⊃ Fx)”
• When a statement in ordinary English has a complex antecedent, we can also run into paraphrasing difficulties:

• How should we paraphrase: “He is attracted to all smart and capable women”? 

  **Option 1**: “(∀x)(Wx · (Sx v Cx) ⊃ Ax)”
  **Option 2**: “(∀x)(Wx · Sx · Cx ⊃ Ax)”